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Review

Human behaviours in evacuation crowd dynamics: From modelling to "big data" toward crisis management

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Abstract

This paper proposes an essay concerning the understanding of human behaviours and crisis management of crowds in extreme situations, such as evacuation through complex venues. The first part focuses on the understanding of the main features of the crowd viewed as a living, hence complex system. The main concepts are subsequently addressed, in the second part, to a critical analysis of mathematical models suitable to capture them, as far as it is possible. Then, the third part focuses on the use, toward safety problems, of a model derived by the methods of the mathematical kinetic theory and theoretical tools of evolutionary game theory. It is shown how this model can depict critical situations and how these can be managed with the aim of minimizing the risk of catastrophic events.

Keywords: Crowd dynamics; Human behaviour; Complexity; Kinetic theory; Stress propagation; Decision making

1. Plan of the paper

The study of human crowds is a challenging interdisciplinary research field that is motivated not only by the difficulties generated by the complexity features typical of large living systems, but also by the benefits that the study of these systems can bring to society. Indeed, the study involves challenging analytic and computational problems, generated by the derivation of models and by their application to practical cases, while the support that models can give to decision making in critical situations can contribute to the safety of citizens.

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The scientific community agrees that an interdisciplinary approach is necessary to tackle the aforementioned conceptual difficulties. The benefit for our society can be important considering that the optimisation of pedestrian flow can reduce the time spent in non-productive activities; ergo reducing the cost of transportation and reducing pollution. Moreover, critical cases, such as sudden and rapid evacuation through complex venues, can occur and affect the safety of walkers. For example, stress induced by the perception of danger, can invoke certain behaviours that may result in dangerous dynamics. Realistic modelling and simulation of crowd behaviours could help to mitigate such risks and lead to significant benefits for society.

For example, simulations can be used to gain insights into possible problems regarding the evacuation of public buildings, metro stations, and ships early in their design phase and/or understanding of how to guide crowd movement in different situations. What is more, pedestrian dynamics is interconnected to several other fields ranging from engineering and architecture to socio-psychology. Our paper specifically focuses on the latter with special attention to account for human behaviours in crowd modelling and their possible support to crisis managing.

The existing literature on general topics of mathematical modelling is reported in some survey papers, which offer different view points and modelling strategies to applied mathematicians in a field where a unified, commonly shared approach does not yet exist. In more detail, the review paper [53] presents and critically analyses the main features of the physics of crowds viewed as a multi-particle system and focuses on the modelling at the microscopic scale for pedestrians undergoing individual based interactions. The survey [61] introduces the modelling at the macroscopic scale, by methods analogous to those of hydrodynamics, where one of the most challenging conceptual difficulties is in the understanding of how the crowd, viewed as a continuum, selects the velocity at which pedestrians move. The survey [24] proposes the concept of the crowd as a living, hence complex system, and subsequently the search of mathematical tools suitable to take into account the complexity features of the system, as far as possible. Scaling problems and mathematical aspects are treated in the survey [18] and more recently in the book [38]. A critical overview of crowd modes is proposed in [44,103].

This literature indicates that the classical modelling approaches can be developed at three observation and representation scales. In detail:

• The *microscopic description*, which refers to individually identified entities, while the overall state of the system is delivered by individual position and velocity of pedestrians. Mathematical models are generally stated in terms of systems of ordinary differential equations.

• The *macroscopic description* is such that the state of the system is described by gross quantities, namely density, linear momentum, and kinetic energy, regarded as dependent variables of time and space. These quantities are obtained by local average of the microscopic state, while models are stated by systems of partial differential equations.

• The *mesoscopic description* is based on kinetic theory methods, where the microscopic state of pedestrians is still identified by the individual position and velocity, however their representation is delivered by a suitable probability distribution over the aforementioned microscopic state. Models describe the evolution of the said distribution function by means of nonlinear integral–differential equations.

It is known that none of the aforesaid scaling approaches are fully satisfactory. In fact, known models at the microscopic scale do not account for multiple interactions and it may difficult, if not impossible, to use data from microscopic observations to infer the crowd dynamics in a different but similar situation. On the other hand, the heterogeneous behaviour of pedestrians get lost in the averaging process needed to derive the macroscopic models which therefore totally disregard this important feature. Mesoscale models appear to be more flexible as they can tackle the previously mentioned drawbacks, but additional work is needed to develop them toward the challenging objectives treated in this paper.

In particular, a multiscale approach is required, where the dynamics at the large scale needs to be properly related to the dynamics at the small scales [12,16,37], hyperbolic and/or parabolic [14,15]. In detail, individuals represent the microscopic scale, their psycho-mechanical strategy is the sub-microscopic scale, while collective behaviours are observed at the macroscopic scale. Recent developments in kinetic theory (mesoscopic scale) lead to this challenging target, as we shall see in the next sections. This result is achieved by introducing, in addition to the mechanical variables, further internal variables in the microscopic state to account for the behavioural interactions between pedestrians. Then these variables can be modified by interactions and have an important influence on the dynamics. The concept of sub-microscopic scale is properly presented in [100], where the authors refer the development of the walking strategy to individual, hence heterogeneous, minds.

Moreover, an important aspect to be taken into account is that emerging behaviours are often related to large deviations although the qualitative behaviours are often reproduced. This feature is related to the fact that small deviations in the inputs create large deviations in the output. Some of these extreme events are not easily predictable, however a rational interpretation can sometimes explain them once they have appeared. The use of the term "black swan" is a metaphoric expression used by Taleb [86] to denote these events. The search for large deviations in crowd dynamics has motivated various empirical studies on this challenging topic, for instance [54,70,84,93].

Although the literature in the field is rapidly growing and is already vast, far less developed are the contributions related to validation dynamics and evacuation processes. Nevertheless, some important contributions need to be mentioned such as the review paper [99]. This reference gives an important contribution to the process of decision support to safety of human crowds in crisis situations and provides a valuable support to our paper. This point can be stressed by quoting a few sentences from [99], selected, among several ones, as specifically pertinent to the aims of our paper:

The importance of understanding human behaviour in crowds is undisputed. It is required for ensuring that proper support can be given to crowd managers in preparation and during crowd event.

Crowd management practice involves accessing and interpreting a wide variety of information sources, predicting crowd behaviours as well as deciding the use of a range of possible, highly context-dependent intervention mechanisms.

We argue that the lack of adequate decision-support is partially due to the status of the majority of current crowd models.

The reasoning above indicates that the overall field needs new ideas to be developed in suitable research programs. Accordingly, this essay is devoted to a critical analysis of the literature concerning crowd modelling in real evacuation dynamics focusing on a support to crisis situations, where modelling and computations are only the first step of a challenging path leading to the improvement of the safety and well-being of citizens. The hallmarks of this path are as follows:

1. *Understanding the main features of a human crowd viewed as a "social" hence complex, system;*
2. *Strategy by which mathematical sciences can contribute to understand the behavioural dynamics of crowds;*
3. *How simulations can be obtained to depict the dynamics through complex venues;*
4. *Understanding how the crowd behaves in extreme situations such as stress induced by perception of dangerous situations;*
5. *What has been done and should be done to respond effectively to crisis situations.*

These topics are motivated by research activity, which is also organized within large research projects. As an example the project [47] of the European Union is treated in the next sections, where each of them corresponds to the aforementioned key problems.

Section 2 refers to the first hallmark with the aim of selecting the most important features of crowds viewed as living systems. The contents is presented in two steps, where the first one refers to normal conditions, while the second one focuses on understanding crowd behaviours in extreme situations such as evacuation. The contents are presented at a qualitative level, namely without analytic formalisation. However, all reasonings offer the conceptual background toward the strategic objective of designing mathematical models suitable to depict the complexity features of the crowd and to reproduce empirical data within reasonable bounds of accuracy.

Section 3 transfers the concepts proposed in Section 2 into a strategy to derive models suitable to capture the aforementioned features. The approach is that of the so called "mathematical approach of the kinetic theory of active particles" [23], where the overall state of the system is represented by a probability distribution over the microscopic states, while interactions, which are nonlocal and nonlinearly additive, are modelled by theoretical tools of evolutionary stochastic game theory. A methodological approach toward the validation of models is proposed, which also accounts for the decision making process to improve human safety. The contents are limited to a qualitative framework leaving the mathematical formalisation to the next section.

Section 4, after a survey on the existing literature on crowd modelling by kinetic theory methods, reports the essential analytic and computational knowledge. This can be of interest for applied mathematicians active in this challenging research field. Thus, the presentation of the preceding sections, that was mainly focused on qualitative issues, meet the formalisation which is necessary for the simulation of real evacuation dynamics. Modelling requires advanced mathematical tools induced both by large dimensions and the complexity features of crowds viewed as a living system. The search of oversimplified approaches generally neglects important features that can play an important influence on the overall dynamics.

Section 5 presents some simulations corresponding to a complex situation. The aim consists in showing the type of results that can be achieved and could contribute to decision making to support crisis; the term "crisis management" is also often used [104]. A detailed analysis of the influence of stress conditions on crowd dynamics is performed by focusing both on the computation of evacuation time and on excessive concentration of individuals.

Section 6 focuses on the main objective of this essay, namely the support that can be given to crisis management for greater situational awareness of crowd dynamics in evacuation situations, leading to a safer process. A critical analysis of the knowledge in the field looks ahead to perspectives toward a deeper understanding of the complex systems under consideration of safety objectives. An important topic, treated in this section, is the design of predictive engines suitable to support the aforementioned decision process. A predictive engine can exploit large databases by using real dynamics and simulations based on the modelling approach reviewed in this paper.

Section 7 proposes a critical analysis of the state of the art as it has been reviewed in our paper and proposes some research perspectives.

It is worth stressing that the whole contents of this paper rely on the concept that a crowd is a living system. Hence, human behaviours have to be taken into account both in the modelling and in crisis management. An interdisciplinary approach appears to be a necessary feature of this paper. Such a need has suggested to split the modelling sections in two parts which deal with general concepts and formalised equations. Therefore, the readership can be broad as the devoted mathematician can intensively focus on Section 4 and related bibliography, while the reader interested in applications can skip over this section and go directly to the following sections.

2. Human crowds as a large living system in evacuation dynamics

The dynamics of a crowd, as already mentioned, cannot be simply confined to mechanical and deterministic causality principles. In fact, the heterogeneous behaviours of pedestrians and their social dynamics can have an important influence over the dynamics and, in particular, in the strategy they use to achieve a certain objective of their movement in interactions with other pedestrians [20]. This strategy is not simply an individual one, it depends on the collective one which, due to nonlocal interactions, can find a consensus toward a commonly shared strategy.

This section tackles the first key problem:

Understanding the main features of a human crowd
viewed as a "social" hence complex system.

Let us now consider the assessment of the most important complexity features of a crowd viewed as a living system within the framework that our society is a complex system [8,11,73]. The general strategy proposed in [23] is that the mathematical approach to modelling of living, hence complex, systems should take into account their features. These general considerations should be focused on the specific field of application treated in this paper, namely the modelling of crowds in evacuation dynamics [1,7,12,13,20]. In particular, evacuation dynamics shows the appearance of special stress conditions. Some stress conditions can be amplified in special venues such as lively foot-bridges [90,91]. Contributions to understand the psychology of a crowd are, selected among various ones, the following [2,32,33,42,98], where stress can end up with panic [66] and even with aggressive behaviours [62].

Bearing all above in mind, let us give, referring to [55], a possible definition of how a crowd can be defined:

Definition of crowd: *Agglomeration of many people in the same area at the same time. The density of people is assumed to be high enough to cause continuous interactions, or reactions, with other individuals,*

while, according to [77] evacuation dynamics can be defined as follows:

Evacuation: *Physical movement of people of a temporary nature, that collectively emerges in coping with community threats, damages or disruptions.*

This definition deserves additional study as it should be related to the scaling problem and, as far as it is possible, to dimensionless numbers suitable to provide a quantitative interpretation suitable to distinguish different flow regimes. Waiting for such a formalisation and, revisiting [20], we look at the following complexity features:

1. ***Ability to express a strategy:*** Walkers are capable to develop specific strategies, which depend on their own state and on that of the entities in their surrounding environment. Different strategies can appear in the dynamics. Examples include pedestrians who move toward different directions, and a crowd in a public demonstration with a small groups of rioters, whose aim is not the expression of a political–social opinion, but instead to create conflict with security forces.
2. ***Heterogeneity and hierarchy:*** The ability to express a strategy is heterogeneously distributed, referring to both the differences in walking abilities, and also to social expressions. This feature can include a possible presence of leaders, who aim to drive the crowd to their own strategy. Leaders can contribute, in evacuation dynamics, to drive walkers toward appropriate strategies including the selection of optimal routes among the available ones.
3. ***Nonlinear and nonlocal interactions:*** Interactions are nonlinearly additive and involve immediate neighbours, but also distant individuals. Interactions refer both to mechanical and social dynamics and include those with the external environment and the venue, where the walkers move. A key example is given by the onset and propagation of stress conditions, which may be generated in a certain restricted area and then diffused over the whole crowd. These conditions can have an important influence over dynamical behaviours of walkers [56].

Of course, additional features could be mentioned, but the selection has been limited to a minimal number that, according to the authors' bias, should be included in the modelling approach. These features hold true in all physical situations. However some of them can be amplified in evacuation dynamics. In fact, in certain conditions, for example overcrowding, avoiding congestion or following others, do create emergent behaviours which are not observed during normal conditions. This topic has been widely and carefully investigated by Helbing and coworkers [54–56]. Summarizing the main features:

- Unanticipated and unintended irregular motion of individuals into different directions due to strong and rapidly changing forces in crowds of extreme density;
- The so called *faster-is-slower* effect, namely increase of the individual speed but toward congested area, rather than the optimal directions, which corresponds to an increase of evacuation time;
- Breaking of cooperative behaviours due to reactions to an event which, in several cases, creates stress.

The above rationale has to be properly related on the understanding of crowd psychology and of the way it affects the dynamics of interacting individuals. Dealing with this challenging, but also fascinating topic [64], goes far beyond the stream of this survey. Indeed, it deserves an additional survey. Some concise ideas are given first on general issues and, subsequently, focusing on the specific topic of this paper.

An important reference is the PhD Thesis [98], more in detail Chapter 2, where it is observed that the first essays on this topic looked at crowd behaviours as a sequence of rational choices, while more recently the so called "irrational behaviors" have been accounted for. Indeed, this is not a peculiarity of crowd psychology as it is witnessed in several fields of soft sciences as observed in [3] for economical and social sciences and in [19] for biological sciences.

Going into more details and specifically referring to Chapter 2 of [98], the author indicates five so called "myths" [82] that can be viewed as an important feature of crowds:

- **Irrationality:** Accounting for the idea that individuals in a crowd lose rational thought;
- **Emotionality:** Individuals in a crowd become more emotional up to shift to riot behaviour; It is to be noted that crowds were seen as an equivalent of riots.
- **Suggestibility:** Individuals in a crowd are more likely to obey or imitate;

- **Spontaneity:** Conveys the idea that in a crowd violence occurs more suddenly;
- **Anonymity:** Individuals in a crowd feel more anonymous;
- **Uniformity/Unanimity:** Conveys the idea that all individuals in a crowd act in the same way.

The author discusses about the use and misuse of term "irrational" to be viewed is often used when people are not behaving in what is seen as the most effective way to achieve a goal, like fleeing out of a building while not following the emergency exits.

The main point, according to the authors bias, is that the all above behaviours are present in a crowd in an heterogeneous way and that specific circumstances, such as evacuation in stress conditions enhance some of them. Therefore, the modelling approach should include all of them, heterogeneity, as well as the heterogeneous behaviour of individuals and the growth of some of them also induced by collective learning [30].

All aforementioned features have to be accounted for by the mathematical tools developed to model the dynamics of a crowd. Subsequently, computational simulations can provide a forecast of the movement of crowds, which can be applied to evacuation processes. This challenging and strategic objective needs not only advanced mathematical tools, but also a deep understanding of human behaviour and the use of technological devices to detect the main features of the crowd. The output delivered by computational models can hopefully support the decision making process needed to tackle crisis situations.

3. Towards a modelling and validation strategy

This section proposes a methodological approach to modelling and validation of crowd dynamics, thus tackling the second key problem proposed in Section 1, namely

Strategy by which mathematical sciences can contribute
to understand the behavioural dynamics of crowds.

The modelling strategy is presented at a qualitative level leaving the mathematical formalisation to Section 4. The contents are referred to the behavioural analysis proposed in Section 2 and the concepts of behavioural crowd dynamics proposed in paper [20]. In fact, pedestrians develop their own dynamics based on an individual interpretation from that of other individuals. As already mentioned, they develop a strategy, which is heterogeneously distributed and which depends on several factors to be included in the modelling approach. These reasonings lead to introduce the already mentioned concept of *behavioural dynamics*. Bearing this in mind, let us anticipate some terminology and some preliminary ideas of the approach that will be developed hereinafter.

- ***Scaling:*** The complexity features of the crowd require a representation which accounts for the heterogeneous behaviour of walkers as well as the difficulty of their deterministic identification. Therefore it is quite natural looking at suitable developments of kinetic theory and statistical dynamics. Hence, the *meso-scale* representation is chosen. Moreover, the approach looks at the so-called kinetic theory for active particles precisely developed to model large systems of interacting entities [23].
- ***Functional subsystems:*** The overall system is subdivided into groups of pedestrians, called *functional subsystems*, who share common "mechanical" features, namely walking to the same direction (different for each group). This subdivision can also include the presence of leaders who operate to drive the evacuation dynamics toward the most appropriate routes.
- ***Representation:*** Each functional system is described by a *probability distribution over the microscopic state of pedestrians*, namely of the variable deemed to define their individual physical state. This is performed according to the hallmarks of a systems theory of social systems introduced in [3] based also on [22,43]. The probability distribution can account for walking with different abilities.
- ***Microscopic state:*** Pedestrians, namely the *micro-system*, are viewed as *active particles*, that have the ability of expressing a their own strategy, called *activity*. This ability can differ for different groups in the same crowd, as it is understood that the activity is heterogeneously distributed.

- ***Interactions at the microscopic scale:*** Interactions are modelled by theoretical tools of evolutionary game theory [74], where individual based interactions featuring the classical theory [72] are replaced by population interactions.
- ***From microscopic to collective behaviours:*** Pedestrians can communicate and develop a *social dynamic*. This communication can diffuse emotional state among walkers. Accordingly, they modify both strategy and dynamical rules followed in their dynamics. The output is a collective behaviour which can be observed over the whole crowd.

A general overview of this approach is presented in the survey paper [23], where the basic concepts of stochastic games are introduced. Applications to model crowd dynamics and social systems are proposed in [13] for a crowd in unbounded domain, and [12] for dynamics in complex venues.

Once a model has been derived, its validation needs to be performed. The validation of models basically means verifying their ability to reproduce empirical data, detected in steady flow conditions at a quantitative level and to depict emerging behaviours at a qualitative level in unsteady conditions. This agreement has to be achieved for a suitable choice of the model's parameters.

The validation of crowd models is a challenging topic that, with a few exceptions such as [80,81,83] and a few others, is poorly treated in the literature. The amount of empirical data available is quite limited for developing a detailed validation process. Hence, a strategy should be elaborated to exploit the existing data at the best of the panorama they offer. An additional difficulty is that the greatest part of empirical data sets are available at the macroscopic scale, while the modelling process needs a detailed understanding of the dynamics at the microscopic scale.

Quantitative validation of models will reproduce the features captured empirically using velocity and flux diagrams that are measured against speed in steady flow conditions.

Qualitatively, emerging behaviours observed in experiments, such as the creation of lanes in narrow streets and increasing evacuation time in stressful conditions, need to be reproduced in the model output.

Bearing all above in mind, let us define more precisely the validation strategy according to the following milestones concerning the performance of a model:

1. Ability to capture the complexity features of a crowd viewed as a living, hence complex, system.
2. Models should reproduce, even at a quantitative level, the velocity and fundamental diagrams of crowd traffic. Moreover, features such as the transition from free to congested flow, with possible changes to interaction rules, should be caught at least at a qualitative level.
3. Models should take into account that environmental conditions can determine different observable dynamics (e.g. different velocity and fundamental diagrams).
4. Models should qualitatively reproduce emerging behaviours. In particular they should catch the transition from small to large deviations by means of properly identified parameters. The mathematical approach should also pursue the idea of designing models able to describe, in evacuation dynamics, severe changes in the dynamics of individual interactions and, hence, in the overall crowd behaviour.

Focusing on the fundamental diagrams, it is worth stressing that their artificial insertion into the equations of the model takes computational models far from the real physics of the system. Indeed, velocity diagrams must be reproduced, as a consequence of interactions at the microscopic scale, and not implemented in the model. These diagrams depend also on the quality of the area where the crowd is located. A possible way to address this issue consists of introducing a parameter related to the quality of the venue where walkers move. High values of this parameter correspond to an increasingly better quality of the areas where walkers move.

4. Kinetic theory and stochastic games toward modelling crowd dynamics

The approach to modelling human crowd dynamics has been developed at the three scales already introduced in Section 1, namely microscopic, macroscopic, and mesoscopic. The literature on the first two scales is reported in some survey papers, which offer to applied mathematicians different view points and modelling strategies in a field where a unified, commonly shared, approach does not exists yet. The review paper [53] introduces the main features of the physics of crowd viewed as a multi-particle system and focuses on the modelling at the microscopic scale for walkers undergoing individual based interactions. The survey [61] deals with modelling at the macroscopic scale by methods analogous to those of hydrodynamics, where one of the most challenging conceptual difficulties exists in understanding how the crowd, viewed as a continuum, selects the velocity direction and the speed by which pedestrians move [60]. The surveys [24] and [18] introduce the concept of the crowds as a living, hence complex system and subsequently the search of mathematical tools suitable to take into account, as far as it is possible, the complexity features of the system under consideration. The book [38] provides a review and critical analysis of the existing literature in the field mainly focused at the microscopic and macroscopic scale.

This section shows how the strategy proposed in the preceding section can be applied to derive specific models suitable to describe the specific dynamics studied in this paper. Therefore, we answer to the third key problem:

How simulations can be obtained to depict the dynamics through complex venues.

The need of computational models to optimise crisis management for crowds is clear. Indeed, models and simulations offer a virtual representation of real dynamics that could form an important part of the information available to crisis managers, which in turn could improve citizens' safety.

A presentation of this topic is proposed through four subsections which deal, respectively, with the following topics: (i) Survey models at the different scales referred to the specific mathematical structures to be used for the modelling approach; (ii) Motivations to select the mesoscopic scale and presentation of a specific model as an example to respond to the requirements of modelling evacuation dynamics; (iii) A critical analysis toward further improvements of the modelling approach in view of the crisis managing.

Simulations to test the predictive ability of the computational model evacuation problems are reported in the next Section 5 with special focus on the role of stress that might appear in the onset of crisis situations.

4.1. Scaling and mathematical structures

Let us consider the crowd in a venue with obstacles, inlet and outlet doors. The whole set of boundaries, doors and their localization is denoted, from now on, by Σ, while the quality of the venue where the dynamics occur is denoted by $\alpha \in [0, 1]$, where $\alpha = 0$ stands for very bad quality that prevents the motion, while $\alpha = 1$ corresponds to the best quality which allows walkers to use their highest possible velocity. An example of venue is given in Fig. 1.

Different mathematical structures correspond to each scale. These structures will be examined in the following to select the most appropriate one for simulations of evacuation processes.

• ***Microscale:*** The *microscopic state* is represented, for each i-th walker with $i \in \{1, \dots, N\}$, by position $\boldsymbol{x}_i = \boldsymbol{x}_i(t) = (x_i(t), y_i(t))$ and velocity $\boldsymbol{v}_i = \boldsymbol{v}_i(t) = (v_x^i(t), v_y^i(t))$. The dynamics refer to a large system of ordinary differential equations of the type

$$\begin{cases} \dfrac{d\boldsymbol{x}_i}{dt} = \boldsymbol{v}_i \,, \\ \dfrac{d\boldsymbol{v}_i}{dt} = \mathbf{F}_i(\boldsymbol{x}_1, \dots, \boldsymbol{x}_N, \boldsymbol{v}_1, \dots, \boldsymbol{v}_N; \Sigma, \alpha), \end{cases} \tag{1}$$

where $\mathbf{F}(\cdot)$ is a psycho-mechanical acceleration acting on the i-th walker based on the action of other walkers in his/her visibility/sensitivity zone. This acceleration depends on Σ, as interactions take into account this feature, which is not accounted for by classical particles until these materially collide with walls.

Often the second equation is also written in the following additive form

$$\frac{d\boldsymbol{v}_i}{dt} = \sum_{j=1}^{n} \varphi_{ij}(\boldsymbol{x}_1, \boldsymbol{x}_j, \boldsymbol{v}_1, \boldsymbol{v}_j; \Sigma, \alpha),$$

with obvious meaning of notations.

The main contributions to this approach have been given by Helbing and coworkers [53,55]. Their approach has been applied also to the modelling of crisis situations [56]. An interesting contribution to compute bifurcation problems in transport equations with nonlocal interactions is proposed in [28].

• ***Macroscale:*** The *macroscopic description* is represented by the *local density* $\rho = \rho(t, \boldsymbol{x})$ and the *mean velocity* $\boldsymbol{V} = \boldsymbol{V}(t, \boldsymbol{x})$, which is referred to maximum mean velocity V_M of walkers.

$$\begin{cases} \partial_t \rho + \nabla_{\boldsymbol{x}} \cdot (\rho\, \boldsymbol{V}) = 0\,, \\ \partial_t\, \boldsymbol{V} + (\, \boldsymbol{V} \cdot \nabla_{\boldsymbol{x}})\, \boldsymbol{V} = \boldsymbol{a}[\rho, \boldsymbol{V}; \Sigma]\,, \end{cases} \tag{2}$$

where $\boldsymbol{a}[\rho, \boldsymbol{V}; \Sigma]$ is a psycho-mechanical acceleration acting on walkers.

The acceleration term $\boldsymbol{a}$ refers not to the individual walker, but to the walkers in the elementary space volume $[\boldsymbol{x}, \boldsymbol{x} + d\boldsymbol{x}]$. Also in this case, it depends on Σ as walkers take into account the geometry of the venue in developing their walking strategy. Phenomenological models are needed to describe the acceleration which acts on all individuals in the elementary volume of space due to the surrounding individuals and geometry of the venue. *First order models* use only the first equation closed by a phenomenological model linking the local mean velocity to local density and density gradients, i.e., $\boldsymbol{V} = \boldsymbol{V}[\rho; \Sigma]$.

Derivation of models at the macroscopic scale has been conceptually introduced by Henderson [57] and subsequently transferred to a framework with some analogy to fluid dynamics by Hughes [60,61], where a method is developed to compute walkers trajectories corresponding to a criterion to optimize the search of the exit. Hughes approach has motivated papers devoted to analytic topics such as [4,5]. Later this method has been approached by mean field games and optimal transport theory as documented, among others, by [29,31,48].

The book [38] provides an exhaustive presentation of macroscopic type models, including those derived by conservation of probability measures. Generally, first order models are used with a few exceptions whereby for second order models see [17,29,31,48]. An important problem, not yet solved, is the modelling of the strategy by which pedestrians organise their movement in stressful conditions that can be induced by overcrowding.

• ***Mesoscale:*** The system is described by the probability distribution function over the microscopic state of walkers defined by their position $\boldsymbol{x} \in \Sigma$ and velocity

$$\boldsymbol{v} \in D_{\boldsymbol{v}} \subset \mathbb{R}^2: \quad f = f(t, \boldsymbol{x}, \boldsymbol{v}) =: \quad [0, T] \times \Omega \times D_{\boldsymbol{v}} \to \mathbb{R}_+,$$

such that $f(t, \boldsymbol{x}, \boldsymbol{v})\, d\boldsymbol{x}\, d\boldsymbol{v}$ denotes the number of active particles whose state, at time t, is in the interval $[\boldsymbol{w}, \boldsymbol{w} + d\boldsymbol{w}]$. Macroscopic quantities are obtained by velocity weighted moments. As an example local density and flux are obtained as follows:

$$\rho[f](t, \boldsymbol{x}) = \int_{D_{\boldsymbol{v}}} f(t, \boldsymbol{x}, \boldsymbol{v})\, d\boldsymbol{v}\,, \qquad \boldsymbol{q}[f](t, \boldsymbol{x}) = \int_{D_{\boldsymbol{v}}} \boldsymbol{v}\, f(t, \boldsymbol{x}, \boldsymbol{v})\, d\boldsymbol{v}. \tag{3}$$

The dynamics is obtained by a balance of microscopic entities in the elementary volume of the space of microscopic state. This amounts to equating the transport of f to the net flow (inlet minus outlet) due to interactions. The formal result is as follows:

$$(\partial_t + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}})\; f(t, \boldsymbol{x}) = \big(J^+ - J^-\big)[f, \Sigma](t, \boldsymbol{x}, \boldsymbol{v}), \tag{4}$$

where J^+ and J^- are, respectively, the inlet and outlet fluxes induced by interaction among walkers and between them and the walls, obstacles and walls. Interactions are nonlinearly additive and nonlocal in space. Theoretical tools of evolutionary theory [72], in this case evolutionary [58,74], stochastic [23] games, are used to model interactions.

The literature on modelling by kinetic theory methods is far less developed than the one at the other scales, however various innovative contributions have been proposed in recent years. The hints proposed in [18,24,59] have been developed in [13] for a dynamics in unbounded domains. Paper [20] has shown how the modelling of interactions with boundaries and internal obstacles can be developed. These two papers introduce the concept of *behavioural crowd*, namely of a dynamic which depends on the strategy and behaviours that walkers develop based also on interactions, mechanical and social, with the surrounding walkers. Validation of such models has been undertaken in [21]. A hierarchy of models is studied in [40], which provides an important conceptual framework for further developments.

A common feature at all scales is the modelling of the strategy by which walkers modify their dynamics, respectively at each scale, the individual acceleration, the collective acceleration and the individual modification of velocity. These are similar quantities, as their difference is related to the scaling only, depends on the shape and quality of the venue Σ and α, respectively.

Empirical data [39,69,71,80,81,83] can contribute to modelling the aforementioned terms as well as on the validation of models [21]. However, it is worth mentioning most of the results focus on the so called velocity and fundamental diagrams, namely mean velocity and flux against density, while additional work needs to be developed toward understanding individual behaviours.

Table 1
Walker's decision process. Firstly walkers change the direction of movement and, afterwards, they modify their speed in probability. Walkers who are close to a wall, reduce the velocity component normal to the wall linearly with the distance from the wall itself, keeping the speed constant. Square brackets denote functional dependence.

	Condition	Transition	Probability
Interactions	$\forall \theta_*$	$\theta_* \to \tilde{\theta} = \theta^{(p)}[\rho, \boldsymbol{\xi}]$	1
	$v_* \leq \xi$	$v_* \to \tilde{v} = \xi + \gamma(1 - \tilde{\rho_p}[\rho, \boldsymbol{\xi}])(\gamma \xi_{\text{LIM}} - \xi)$	$\gamma(1 - \tilde{\rho_p}[\rho, \boldsymbol{\xi}])$
		$v_* \to \tilde{v} = v_*$	$1 - \gamma(1 - \tilde{\rho_p}[\rho, \boldsymbol{\xi}])$
	$v_* > \xi$	$v_* \to \tilde{v} = \xi - \tilde{\rho_p}[\rho, \boldsymbol{\xi}]\xi$	$(1 - \gamma C)\tilde{\rho_p}[\rho, \boldsymbol{\xi}]$
		$v_* \to \tilde{v} = v_*$	$1 - (1 - \gamma C)\tilde{\rho_p}[\rho, \boldsymbol{\xi}]$
Boundary	$d_* < d_w$	$\tilde{\theta} \to \theta = \theta^{(w)}$	1
		$\tilde{v} \to v = \tilde{v}$	1

4.2. Selection of the mesoscopic scale and of a mathematical model

Our paper has selected the mesoscopic scale as the most appropriate to capture the complexity features of human crowd which have been presented in Section 2. These features play a key role in the study of evacuation phenomena. In more detail, the heterogeneity of walkers can be taken into account by the representation of the system by a probability distribution. The modelling of interactions by theoretical tools of game theory allows to include all possible trends of walkers interacting with each other and with the venue where they walk. Subdivision into functional subsystems allows the inclusion of different typologies of walkers from leaders to groups that need physical support to evacuate.

Let us consider a crowd which, according to the modelling approach proposed in the preceding section, can be subdivided into n functional subsystems (FSs). The state of the overall system is described by the one-particle distribution functions $f_i = f_i(t, \boldsymbol{x}, \boldsymbol{v})$ with $i = 1, \ldots, n$.

The mathematical model used in the present work has been described in Ref. [21] and reads

$$\begin{aligned}(\partial_t + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}})\; f_i(t, \boldsymbol{x}, \boldsymbol{v}) &= \eta_{\text{A}} \left(\int_{\mathcal{V}} \mathcal{A}[\rho, \boldsymbol{\xi}; \Sigma](\boldsymbol{v}_* \to \boldsymbol{v}) f_i(t, \boldsymbol{x}, \boldsymbol{v}_*)\, d\boldsymbol{v}_* - f_i(t, \boldsymbol{x}, \boldsymbol{v}) \right) \\ &+ \eta_{\text{B}}(\boldsymbol{x}) \left(\int_{\mathcal{V}} \mathcal{B}(\boldsymbol{v}_* \to \boldsymbol{v}) f_i(t, \boldsymbol{x}, \boldsymbol{v}_*)\, d\boldsymbol{v}_* - f_i(t, \boldsymbol{x}, \boldsymbol{v}) \right)\end{aligned} \tag{5}$$

where η_{A} and η_{B} are the interaction rates between walkers and between walker and walls, and $\mathcal{A}$ and $\mathcal{B}$ are the transition probability densities which models the decision process based on which walkers modify their velocity. In Eq. (5), square brackets have been used to denote the functional dependence of the transition probability density $\mathcal{A}$ from the local mean density and velocity. Therefore, in spite of its linear appearance, the proposed crowd model is a strongly nonlinear set of integro-differential equations.

The interaction rate between walkers, η_{A}, is assumed to be constant while it is supposed that walkers interact with walls only when they are sufficiently close to them. Accordingly the interaction rate η_{B} is space dependent. The main features of the the transition probability densities are summarized in Table 1.

Interactions between walkers are assumed [20,21] to modify their dynamics firstly by changing the direction of movement and, afterwards, by modifying the speed

$$\mathcal{A}[\rho, \boldsymbol{\xi}, \Sigma](\boldsymbol{v}_* \to \tilde{\boldsymbol{v}}) = \mathcal{A}_v[\rho, \boldsymbol{\xi}](v_* \to \tilde{v})\mathcal{A}_\theta[\rho, \boldsymbol{\xi}](\theta_* \to \tilde{\theta}) \tag{6}$$

where the velocity has been decomposed in speed and direction $\boldsymbol{v} = \{v, \theta\}$.

Three types of stimuli are assumed to contribute to the modification of walking direction, namely, the desire to reach a defined target, the attraction toward the mean stream and the attempt to avoid overcrowded areas. These are represented by the three unit vectors $\boldsymbol{\nu}_i^{(t)}$, $\boldsymbol{\nu}_i^{(s)}$, and $\boldsymbol{\nu}^{(v)}$, respectively. It is expected that at high density, walkers try to drift apart from the more congested area moving in the direction of $\boldsymbol{\nu}^{(v)}$. Conversely, at low density, walkers head for the target identified by $\boldsymbol{\nu}_i^{(t)}$ unless their level of anxiety is high in which case they tend to follow the mean stream as given by $\boldsymbol{\nu}_i^{(s)}$. Accordingly, the preferred direction is defined as

$$\boldsymbol{v}_i^{(p)} = \frac{\tilde{\rho}\boldsymbol{v}^{(v)} + (1-\tilde{\rho})\dfrac{\beta\boldsymbol{v}_i^{(s)} + (1-\beta)\boldsymbol{v}_i^{(t)}}{\left\|\beta\boldsymbol{v}_i^{(s)} + (1-\beta)\boldsymbol{v}_i^{(t)}\right\|}}{\left\|\tilde{\rho}\boldsymbol{v}^{(v)} + (1-\tilde{\rho})\dfrac{\beta\boldsymbol{v}_i^{(s)} + (1-\beta)\boldsymbol{v}_i^{(t)}}{\left\|\beta\boldsymbol{v}_i^{(s)} + (1-\beta)\boldsymbol{v}_i^{(t)}\right\|}\right\|}, \tag{7}$$

where $\tilde{\rho} = \rho/\rho_{\text{MAX}}$, being ρ_{MAX} the highest admissible packing density, and

$$\boldsymbol{v}^{(v)} = -\frac{\nabla_{\boldsymbol{x}}\rho}{\|\nabla_{\boldsymbol{x}}\rho\|}, \qquad \boldsymbol{v}_i^{(s)} = \frac{\boldsymbol{\xi}}{\|\boldsymbol{\xi}\|} \tag{8}$$

In Eq. (7), $\beta \in [0, 1]$ is a parameter which models the sensitivity to the stream with respect to the search of vacuum and it is supposed modelling, to some extent, the level of anxiety of walkers.

The transition probabilities for angles is thus defined

$$\mathcal{A}_\theta[\rho, \boldsymbol{\xi}](\theta_* \to \tilde{\theta}) = \delta\left(\tilde{\theta} - \theta^{(p)}\right) \tag{9}$$

where the preferred angle of motion, $\theta^{(p)}$, is obtained from Eq. (7) through the relation $\boldsymbol{v}_i^{(p)} = (\cos\theta^{(p)}, \sin\theta^{(p)})$.

For what speed is concerned, we first introduce the perceived density along the direction $\theta^{(p)}$ which reads

$$\rho^* = \rho^*[\rho] = \rho + \frac{\partial_p\rho}{\sqrt{\rho_{\text{MAX}}^2 + (\partial_p\rho)^2}}\left[(\rho_{\text{MAX}} - \rho)\, H(\partial_p\rho) + \rho\, H(-\partial_p\rho)\right], \tag{10}$$

where ∂_p denotes the derivative along the direction $\theta^{(p)}$ while $H(\cdot)$ is the Heaviside function $H(\cdot \geq 0) = 1$, and $H(\cdot < 0) = 0$. According to Eq. (10), it results

$$\partial_p\rho \to \infty \Rightarrow \rho_p \to \rho_{\text{MAX}}, \quad \partial_p\rho = 0 \Rightarrow \rho_p = \rho, \quad \partial_p\rho \to -\infty \Rightarrow \rho_p \to 0.$$

Bearing all above in mind, two cases are distinguished:

- The walker's speed is greater than (or equal to) the mean speed. The walker either maintains its speed or decelerate to a speed ξ_d which is as much lower as density become higher. It is reasonable to assume that the probability to decelerate, p_d, increases with the congestion of the space, the quality of the venue and the anxiety level of the walker measured by the parameters α and β, respectively.
- The walker's speed is lower than the mean speed. The walker either maintains its speed or accelerate to a speed ξ_a which is as much higher as density become lower, the higher is the gap between the mean speed and the preferred speed and the goodness are the environmental conditions. It is reasonable to assume that the probability to accelerate, p_a, decreases with the congestion of the space and with the badness of the environmental condition and the anxiety level of the walker.

Accordingly the transition probability density for the speed reads

$$\begin{aligned} \mathcal{A}_v[\rho, \boldsymbol{\xi}](v_* \to \tilde{v}) = &\{p_a\delta(\tilde{v} - \xi_a) + (1 - p_a)\,\delta(\tilde{v} - v_*)\}\, H(\xi - v_*) \\ &\times \{p_d\delta(\tilde{v} - \xi_d) + (1 - p_d)\,\delta(\tilde{v} - v_*)\}\, H(v_* - \xi), \end{aligned} \tag{11}$$

where

$$\xi_d = \xi - \tilde{\rho}_p\xi, \qquad p_d = (1 - \gamma C)\tilde{\rho}_p, \tag{12}$$

and

$$\xi_a = \xi + \gamma(1 - \tilde{\rho}_p)(\gamma\xi_{\text{LIM}} - \xi), \qquad p_a = \gamma(1 - \tilde{\rho}_p). \tag{13}$$

In Eqs. (12) and (13), $\tilde{\rho}_p = \rho_p/\rho_{\text{MAX}}$, $\gamma = \alpha\beta$ where the parameter α measures the quality of the area in which the crowd is located, and the constant $C < 1$ has been introduced so as to take into account that the probability to decelerate is not naught even if $\gamma = 1$.

The most important feature to be taken into account in modelling the interactions between walkers and walls is its non-locality, as walkers are not classical particles and modify their velocity before encountering the wall. Accordingly, it is supposed that walkers whose distances from the wall, d, are within a specified cutoff, d_w, modify their velocity $\tilde{\boldsymbol{v}}$ to a new velocity $\boldsymbol{v}^{(w)}$, by reducing the normal component linearly with the distance from the wall but keeping the speed constant, that is

$$\boldsymbol{v}^{(w)} = \frac{d}{d_w}\,(\tilde{\boldsymbol{v}}\cdot\boldsymbol{n})\,\boldsymbol{n} + \mathrm{sign}(\tilde{\boldsymbol{v}}\cdot\boldsymbol{t})\left[\tilde{v}^2 - \frac{d^2}{d_w^2}\,(\tilde{\boldsymbol{v}}\cdot\boldsymbol{n})^2\right]^{1/2}\boldsymbol{t}, \tag{14}$$

where $\boldsymbol{n}$ and $\boldsymbol{t}$ are the normal and tangent to the wall.

The transition probability density $\mathcal{B}$ is then defined as

$$\mathcal{B}(\tilde{\boldsymbol{v}} \to \boldsymbol{v}) = \delta\left(\theta - \theta^{(w)}\right)\delta\left(v - v^{(w)}\right), \tag{15}$$

where $v^{(w)} = \tilde{v}$ and $\theta^{(w)}$ is the direction of the velocity $\boldsymbol{v}^{(w)}$ defined by Eq. (14) through the relation $\theta^{(w)} = \boldsymbol{v}^{(w)}/v^{(w)} = (\cos\theta^{(w)}, \sin\theta^{(w)})$.

4.3. Critical analysis

A mesoscopic model has been selected out of the existing literature. It has been validated [21] by using empirical data according to the hallmarks reported in Section 3. In summary, this model has shown the ability to reproduce empirical data available in steady uniform conditions, namely the velocity and fundamental diagrams. In addition, some emerging behaviours have been reproduced such as line fingering in corridors, overcrowding around outlet doors, increase of the evacuation time due to stress conditions. The simulations presented in the next section will show some interesting emerging behaviours in evacuation dynamics. However, despite the success of the model, several problems remain open. Therefore, some of them are brought to the attention of the reader as possible future research perspectives. In detail:

1. *Empirical data* almost always, refer to steady uniform flow conditions and provide an information on macroscopic quantities. On the other hand, the modelling approach needs data on the interaction at the microscopic scale far from steady conditions. In addition, the knowledge on the variety of emerging behaviours is still quite limited. Hopefully, future research activity will provide additional information on empirical data to improve the present state of the art [35,36].
2. *Empirical data on microscopic behaviours* should be addressed to tackle the conceptual difficulty of modelling the dependence of walkers' dynamics on α and Σ, so that a careful modelling of interactions avoid the artificial insertion (as it happens in a great variety of models) of the velocity diagram into the model. Indeed, such a diagram should be an emerging collective behaviour of interactions. Enlightening the link between microscopic and macroscopic dynamics deserves special attention [102].
3. *Social dynamics* in crowds refers to interaction of walkers that exchange their psychological attitude toward a consensus to a common walking strategy. The model takes into account this specific feature focusing on stressful conditions that might reduce safety conditions. An interesting topic, still to be developed, is the diffusion of other types of behaviours. An example is given by the presence of rioters in a democratic manifestation, when they attempt to obtain consensus from the other pacific demonstrators. A deep understanding of collective learning [30] can contribute to modelling social interchanges that can include violent acts [46] and transition into violence due to communications with rioters.
4. *Multiscale problems:* An important topic consists in the derivation of macroscopic models from the underlying description at the microscopic scale has been developed in [12] following the approach previously applied to vehicular traffic [16], while coupling pedestrian traffic to vehicular traffic has been developed in [27]. These multiscale methods tackle the criticisms of the heuristic approach to the derivation of macroscopic models which might not be properly related to the dynamics at the microscopic scale.
5. *Computational problems* should be referred to the use of the simulations. In some cases, such as real time use of simulations, the computational time should be less than or equal the real one, although paying the price of accuracy. Otherwise, models are required to be as accurate as possible according to the validation rules presented

in Section 3. This issue, related to the dynamics of slow and fast thinking [63], that can play an important role in crisis managing.

The critical analysis of this section should also remark that a great deal of activity has still to be done to provide mathematical models useful for safety management. As a matter of fact, some recent papers [68,88,89,100] have well expressed this need. It has been shown that the model in [21] covers a certain amount of needs such as consistency with complexity features, validation based on empirical data, and descriptive ability of stress conditions that appear during evacuation. However, additional work needs to be developed to improve its predictive ability. In particular, two topics can be brought, as possible examples, to the attention of the reader. In detail:

1. The parameter β accounts for stress conditions and plays an important role in the overall dynamics. It can evolve in time, hence it should be treated as an additional internal variable. The modelling of the probability density $\mathcal{A}$ should include also individual modification of β due to interactions. This approach is deemed to model propagation of the stress conditions.
2. An important issue to be accounted for is the possible subdivision in the crowd of democratic individuals and rioters followed by transitions across these two groups. This important dynamics can be described, with some analogy with the modelling of biological mutations [19], by inserting in the games also the probability of transition across the two groups.

Definitely, future research activity will focus on these topics, where the challenging task is the modelling of interactions and their insertion into a mathematical structure. This perspective cannot be confined only to kinetic type models. Future developments of macroscopic and microscopic models would be welcome in addition to the classical problems already mentioned in the preceding subsection. In particular, consistency with the complexity paradigms can be achieved by subdividing the crowd into different functional subsystems to approach a possible description of heterogeneity, while nonlinearly additive and nonlocal interactions should be modelled toward real behaviours in human crowds. The final target consists in the achievement of a unified modelling approach, such that models at the microscopic scale led to the derivation of models at the mesoscopic scale, and asymptotic methods [12] lead to macroscopic (hydrodynamic) models.

Models at all scales should also investigate the conjecture [9], nowadays considered valid for animal swarms, that each individual interacts with a fixed number of other individuals in a swarm rather than with all of them in their interaction domain. The mathematical formalization of this conjecture can be found in [25]. Is it true also for human crowds. Arguably, this is true, but depending on psychological conditions. We do not have empirical data for this topic on crowds. Therefore, this is a request from mathematical sciences to experimentalists active in the search of empirical data on crowd dynamics.

5. Sample simulations in complex venues and impact of stress conditions

This section tackles the fourth key problem and presents some simulations to test the predictive ability of the model presented in Section 4 as well as to enlighten the dynamics in stress conditions.

Understanding how the crowd behaves in extreme situations
such as stress induced by perception of danger.

Computational results are obtained by Monte Carlo particle methods first introduced by Bird [26] and subsequently developed by various authors by adjusting the method to each specific system under consideration, see [10,75] and many others. A computational scheme for the specific application to crowd dynamics have been developed in [21]. Compared to deterministic methods of solution [6,52], the particle simulations provide some important advantages such as the ability to deal with complex geometries and to easily account for sophisticated individual decision processes. It is worth noticing that, although particle methods are computationally very efficient, parallel computing may be necessary to reduce the computational burden. In this respect, the possibility of exploiting the massively parallel architecture of modern Graphics Processing Units would be certainly of interest [49,50].

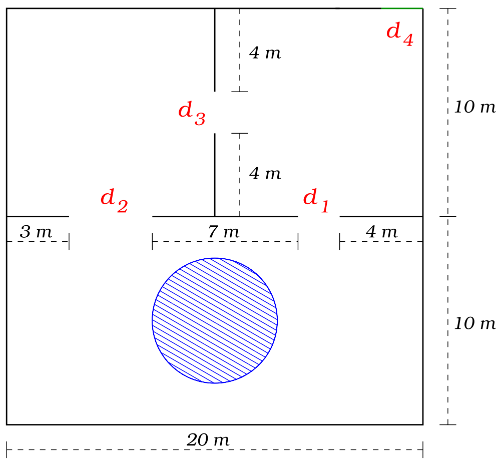

Fig. 1. Geometry of the venue.

Simulations can contribute to crisis management not only by showing that the model can provide an accurate description of the crowd dynamics, but also by verifying how the evacuation time increases under stress conditions and the identification of risk situation due to an excessive concentration of walkers in the same area. Indeed, safety conditions require that local density remains below a safety level. These dynamics depend on venue parameters. Hence, simulations can contribute to optimise the areas of the venue for crowds.

A sketch of the initial conditions and of the geometry of the venue is shown in Fig. 1 consisting of three rooms connected by the doors d_1, d_2, and d_3, while the exit is located in the upper right corner at door d_4. In detail, the following dynamics are considered: *A group of walkers concentrated in a circular area, of radius* $3\,m$*, initially just standing. Then, when an evacuation information is given, the original group symmetrically divides into two groups moving toward the closest between the doors* d_1 *and* d_2*. Walkers who do not get through door* d_1 *before it suddenly closes at* $t = 20s$*, are obliged to change direction and move to door* d_2 *in the attempt to reach door* d_4*.*

The evacuation time corresponding to two different initial conditions is reported in Fig. 2, that shows the ratio between the walkers in the room and their initial number versus time for different density of the crowd and with/without the increase of stress after the incident occurrence. Flow patterns are shown in Fig. 3 for a sample of 50 walkers, with an onset of stress conditions modelled by a sudden increase of the parameter β, when the door closes.

These simulations only cover a small part of the overall study which, thank to several possible conditions and parameter sensitivity analysis, have allowed to draw the following conclusions:

- Independently on the crowd density, at the first stage of the evacuation process, the higher is β, the faster is the evacuation time (dashed lines are above solid lines). This is not unexpected since the walkers mean velocity increases with β. However, in the long run stressful conditions increase the evacuation time (dashed lines are below solid lines).
- For equal stress conditions, the higher the crowd density, the slower the evacuation process (black lines are below red lines). Indeed, areas with high density arise from the dynamics, thus increasing the congestion level and consequently reducing the local mean velocity.
- Modifications of the venue can be studied with the aim of improving the quality of the evacuation dynamics, mainly to reduce high concentration of crowds.

6. Decision making toward information management and crisis response

This section takes advantage of the computational modelling and analysis of stress conditions to deal with the fifth key problem posed at the beginning of this paper:

What has been done and should be done to respond effectively to crisis situations.

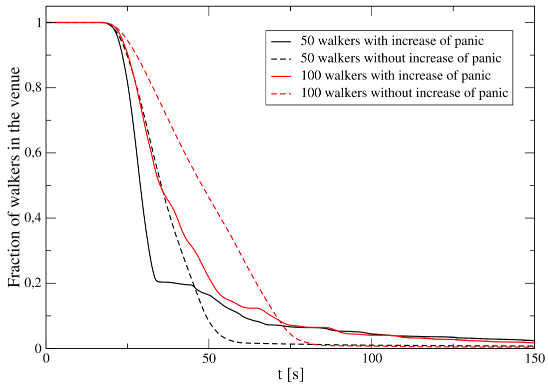

Fig. 2. Fraction of walkers in the room versus time. Evacuation of a crowd composed of 50 (black lines) and 100 (red lines) walkers, when the parameter β modifies from 0.5 to 0.8 (solid lines) and remains at the constant value of 0.5 (dashed lines). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

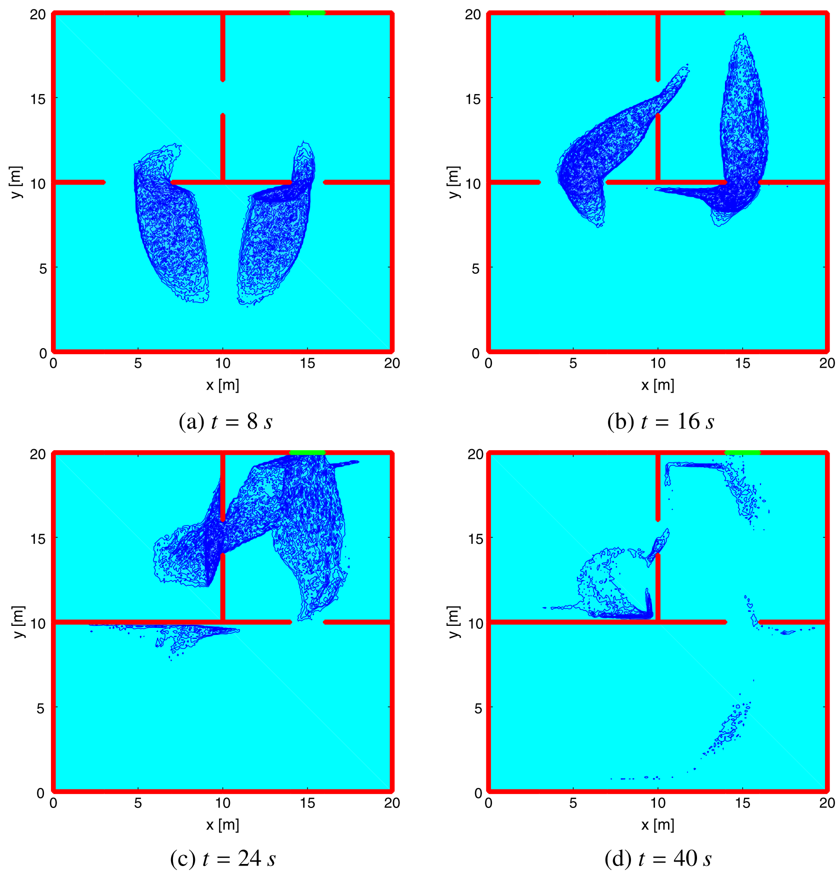

Fig. 3. Density contour plots of a crowd composed by 50 walkers evacuating the room at different instants of time. Panic modifies β from 0.5 to 0.8 when the door closes.

Therefore, this section is devoted to understand the needs of decision makers and consequently, how the information delivered by simulations can be used to support the decision making during a crisis situation. As we have seen the dynamics of evacuation can generate a crisis situation whenever safe conditions appear to be lost, generally when the dynamics generate situations of overcrowding. Then, support to the crisis manager is needed to select the most appropriate strategy to reduce danger. Several technical difficulties need to be tackled. For instance, in most cases, decision making has to be developed in a very short time [89] and the information is not complete [79] and various possible alternatives can be considered [45]. The support that can be given to managers of evacuation processes covers various aspects. The first one consists of general protocols, optimized by past experience, which can be applied in a variety of situations, namely for different venues, generally specialised for similar environments, e.g. buildings, under-grounds, airports, and so on, which require somewhat different strategies. More refined protocols can be obtained by further specialisation for well-defined venues.

Two aspects of the general problem can be enlightened:

1. *Training crisis managers* by visualisation of a big number of simulations stored in a database corresponding to different venues, crowd features and actions. Beyond visualization, managers can work out the most appropriate actions to dangerous situations.
2. *Selection of the most appropriate actions* during an evacuation process.

The first step needs the design of a *database* repository of big data, while the second step needs the design of a *predictive engine* to support the aforementioned actions. The literature on database repositories of big data is rapidly growing in this century as witnessed by the report [34] and by paper [41], as well as by the reports posted in the web-sites [94–97].

Simulations to be stored in the database need to be validated also referring to the specific venue, where the dynamics occur [65,78]. Simulations should refer to evacuation dynamics [1,76,87], and need to be specifically related to support crisis [89,92,101].

The conceptual problem consists in understanding how these data can be used for predictive purposes or for model validation. The various methods to treat these large amounts of data still need to be properly developed to define an emerging data science which aims at improving the decision making process toward cost reductions and reduced risk. Therefore, approaches toward the interpretation of large data stored in databases should go beyond the technical problem of data compression and their statistical interpretation. More sophisticated is the problem of designing the predictive engine as different situations have to be compared and the selection of the most appropriate action requires an appropriate selection of the distance (metric) between different dynamical systems. In fact, such a distance can provide the correct information to decide how far a simulation is close to the real dynamics observed by the crisis manager. Decision makers are being forced to weigh the interaction of numerous factors that characterize the situation and the corresponding most effective decision [85].

Other aspects for decision support consist of information management, which can be split into different parts:

- Accessing information: Virtual representation of real crowd dynamics might deliver important information for crisis managers. Although evacuation strategies depend on several factors a real time forecast of the crowd movements (strategies) might help the crisis manager to identify risk situations due to changing crowd conditions;
- Interpreting information: Human behaviour has to be taken into account in decision making/crisis response;
- Prediction of crowd behaviour: Decision support could be optimised by using crowd model results. To trust the modelling, the decision maker should know what is inside and should understand the different parameters, their level of uncertainty, and the assumptions, for example on social interchanges.

Crises management is a challenging topic, which currently involves scientific dispute [67] and it is difficult, at present, to find a commonly shared theory. Focusing specifically on the use of crowd simulation, the approach can take advantage of a recent paper [3], where a systems approach to social dynamics is proposed. This is an approach based on the following sequence of actions that can be transferred to the specific class of problems treated in our paper as follows:

1. Characterization of the evacuation venues;
2. Characterization of the main features of the crowd;
3. Implementation of a number of possible simulations corresponding to both aforementioned classifications;
4. Implementation of a number of simulations corresponding to possible actions to control the evacuation process;
5. Scoring the output of the dynamics mentioned in Item 4;
6. Define a metric to compare a real situation to those stored in the database;

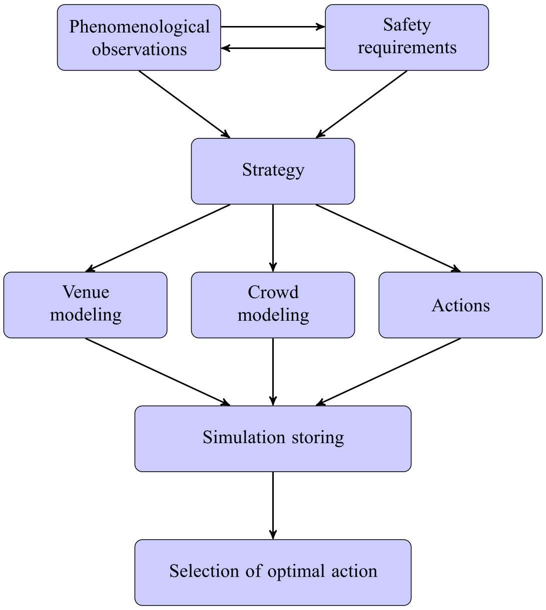

Fig. 4. Rationale of the support process.

7. Select a number of simulations close to the real situation and select, out of them, the most appropriate action based on a weighted combination of the score and the metric distance defined in Item 6.

Namely, the best strategy is selected by the simulation which shows the highest evaluation from a weighted linear superposition of the similarity evaluation and the score given to the output of the selected simulation. The selection by similarity rules should account for likelihood principles [51]. The rationale of the support process, namely the core of the predictive engine, is shown in Fig. 4.

This process succeeds to provide a technical response to the two objectives defined in this section so that the heuristic approach based on personal bias is avoided. The reliability of the learning machine depends on the validity of the simulations therein stored. Therefore, future research activity should be intensively addressed to improve the modelling approach also in connection with the design of engines to support decision making. Another aspect to address for future research is the reliance of decision makers before or during a crisis upon the technical modelling concepts and results. The conceptual difference of this approach compared with [104] is that it can provide a real time interpretation of ongoing crowd dynamics rapidly related to optimised possible decision processes.

7. Critical analysis

The overview on crowd dynamics and safety problems presented in this paper has shown that the literature in the field can give valuable contributions to the crisis management of human crowds in evacuation situations. However, it is worth stressing that several problems are still open and need further research activity. Some perspectives can be given out of said overview and critical analysis.

Without claim of completeness, some remarks can be referred to the three sentences quoted, from [99], in Section 1:

1. *The importance of understanding human behaviour in crowds is undisputed. It is required for ensuring that proper support can be given to crowd managers in preparation and during crowd event:* This important hint indicates that understanding social and dynamical behaviours of a crowd is the absolutely necessary basis for any decision process related to safety. The problem consists not only of acquiring this type of information, but also support practical decision making. Our paper has put in evidence that any approach should consider the crowd as a living, hence complex, system. Hence, understanding the complexity features of human crowd is very important also in designing computational models.

2. *Crowd management involves accessing and interpreting a wide variety of information sources, predicting crowd behaviours as well as deciding the use of a range of possible, highly context-dependent intervention mechanisms:* Indeed, a broad variety of information sources is very important. Here we simply stress that the design of the predictive engine can contribute to select the available information. However, data to be inserted should be properly assessed. Otherwise, the information can be even misleading.
3. *The authors agree that decision support can be aided by the inclusion of relevant, validated and practical use of crowd models; and the existing literature on crowd modelling can only partially support crisis management.* As shown in this paper, further parameters could be modelled to increase the relevance and accuracy of models used for this purpose. Indeed, new modelling techniques are often required to achieve this as proposed in this paper. These techniques can account for some of the identified enhancements, though we claim to have covered them exhaustively.

Partially positive answer in the last issue indicates that future research activity on crowd modelling should focus on a deeper integration of psychological and behavioural features in models. This effort can be supported by empirical data on crowd detection specifically related to social behaviours.

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